Solving Double Inequalities

The solution to a system of two inequalities in one variable consists of all values of the variable that make each inequality in the system true. A system \( f(x) \geq a, f(x) \leq b \), where the same expression appears on both inequalities, is commonly referred to as a “double” inequality and is often written in the form \( a \leq f(x) \leq b \). Be certain that both inequality signs are pointing in the same direction and that the double inequality is only used to indicate an expression in \( x \) “trapped” in between two values. Also \( a \) must be less than or equal to \( b \) in the inequality \( a \leq f(x) \leq b \) or \( b \geq f(x) \geq a \).

**Example**

Solve a double inequality, using graphical techniques.

\[
\begin{align*}
2x - 5 & \geq -1 \\
2x - 5 & \leq 7
\end{align*}
\]

**Before Starting** There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

**Step & Key Operation**  

1. Enter \( y = -1 \) for \( Y1 \), \( y = 2x - 5 \) for \( Y2 \), and \( y = 7 \) for \( Y3 \).

   
   Y= \[ (-) \] 1 ENTER

   \[ 2 \] \[ \leftarrow \] \[ - \] 5 ENTER 7

2. View the lines.

   GRAPH

3. Find the point of intersection.

   2nd F CALC 2

**Display**

The “double” inequality given can also be written to \(-1 \leq 2x - 5 \leq 7\).

**Notes**

y = 2x - 5 and y = -1 intersect at (2, -1).
### Step & Key Operation | Display | Notes
--- | --- | ---
4 | Move the tracer and find another intersection. | ![Graph showing two lines intersecting at (6,7)](image) | y = 2x - 5 and y = 7 intersect at (6,7).

5 | Solve the inequalities. | The solution to the “double” inequality -1 ≤ 2x - 5 ≤ 7 consists of all values of x between, and including, 2 and 6 (i.e., x ≥ 2 and x ≤ 6). The solution is 2 ≤ x ≤ 6.

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Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often difficult to solve algebraically. The EL-9900 allows the solution region to be indicated visually using the Shade feature. Also, the points of intersection can be obtained easily.