SHARP

ADVANCED D.A.L.

SCIENTIFIC CALCULATOR TEACHER'S GUIDE

EL-531RH | JULY 1999

Contents

Teacher's Guide Part 1 has already been completed. This guide presents Part 2 beginning on page 39 (marked with NEW). We encourage you to put this guide to good use.

Introduction

The use of calculators as a classroom teaching tool is becoming more and more popular. Contrary to the belief that their use encourages dependency and inhibits the development of mental skills, research has proven that calculators are highly unlikely to harm achievement in mathematics and using them can actually improve the students' performance and attitude.* Calculators allow students to quickly generate large amounts of data from which patterns can be spotted, and predictions can be made and tested. This is an important aspect of the development of mental methods of calculation. Therefore, priority must be given to create new ways to exploit the potential of the calculator as an effective learning tool in the classroom.

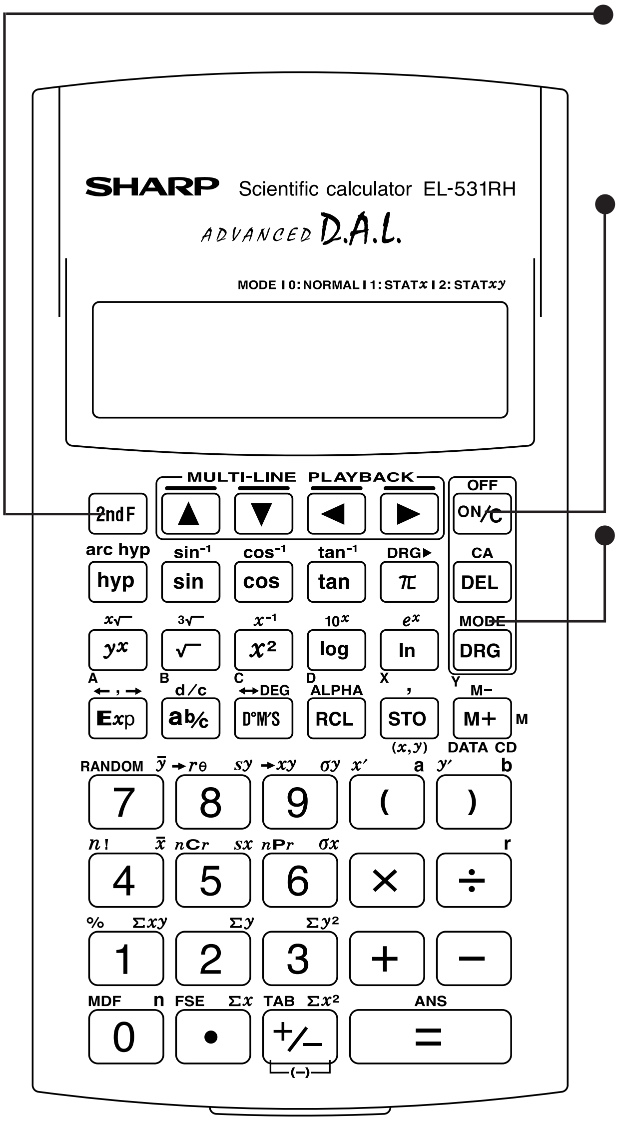
This Teacher's Guide presents several classroom activities that make use of Sharp scientific calculators. The purpose of these activities is not to introduce the calculator as a device to relieve the burden of performing difficult calculations, but rather to develop the students understanding of mathematical concepts and explore areas of mathematics that would otherwise be inaccessible. Mental methods should always be considered as a first resort when tackling calculations introduced in these activities. The development of trial and improvement methods are supported by the activities as well. We hope you will find them interesting and useful for reinforcing your students' understanding of mathematical concepts.

* Mike Askew & Dylan Williams (1995) Recent Research in Mathematics Education HMSO

How to Operate

≈Read Before Using≈

1. KEY LAYOUT



2nd function key

Pressing this key will enable the functions written in yellow above the calculator buttons.

ON/C, OFF key

Direct function	2nd function
ON/C	2ndF OFF
<Power on>	<Power off>

Written in yellow above the ON/C key

Mode key

This calculator can operate in three different modes as follows.

<Example>

[Normal mode]

2ndF MODE 0

•Mode = 0; normal mode for performing normal arithmetic and function calculations.

[STAT-1 mode]

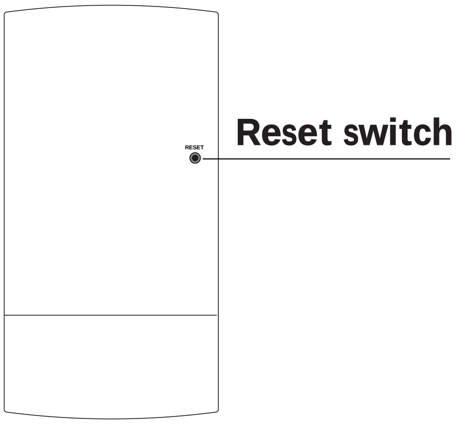
2ndF MODE 1

•Mode = 1; STAT-1 mode for performing 1-variable statistical calculations.

[STAT-2 mode]

2ndF MODE 2

•Mode = 2; STAT-2 mode for performing 2-variable statistical calculations.



2. RESET SWITCH

RESET

If the calculator fails to operate normally, press the reset switch on the back to reinitialise the unit. The display format and calculation mode will return to their initial settings.

NOTE:

Pressing the reset switch will erase any data stored in memory.

Number Bowling

Junior high school

Objective

Read whole numbers and understand that the position of a digit signifies its value.
Understand and use the concept of place value in whole numbers.

Explanation of the activity

Think of a 3-digit number and enter it into your calculator.
Pretend each digit is a “bowling pin.”
Knock down each pin one at a time, so that your calculator display shows 0.

A: Using subtraction
B: Using addition

Using the calculator

Calculator functions used: Subtraction, addition, last answer memory

A: Using subtraction

Press the following buttons and then start operation.

`ON/C` `2ndF` `MODE` `0`

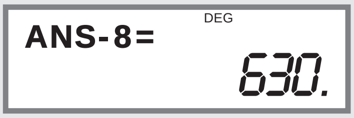
(1) Enter a 3-digit number.

638 `=`

638=
638.

(2) Knock down one digit, or “pin”; i.e. change the last digit to a 0.

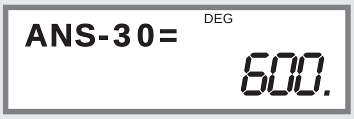
`−` 8 `=`



ANS-8=
630.

(3) Knock down the next pin; i.e. change the tens column digit to 0.

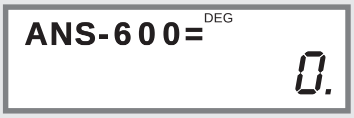
`−` 30 `=`



ANS-30=
600.

(4) Knock down the pin of the hundreds column.

`−` 600 `=`



ANS-600=
0.

Number Bowling

Junior high school

B: Using addition

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

(1) Enter a 3-digit number.

638 =

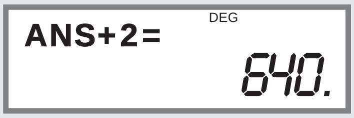
638=

DEG

638.

(2) Knock down one digit, or pin; i.e. change the last digit to a 0, except this time, do so by adding a number to the last digit to make it 0.

\+ 2 =



ANS+2=

DEG

640.

(3) Knock down the next pin; i.e. change the tens column digit to 0.

\+ 60 =

ANS+60=

DEG

700.

(4) Knock down the pin of the hundreds column.

\+ 300 =

ANS+300=

DEG

1000.

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity is a good game for students to play in pairs.
One student enters a number in the calculator, and the other student has to knock each digit, or "pin," down.

Example:

638 - 8 = 630
630 - 30 = 600
600 - 600 = 0

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It is important for students to talk about what they are doing and use the appropriate language, for example: "six hundred and thirty, minus thirty, equals six hundred." Students should be challenged to vary the starting point; i.e. sometimes starting with the hundreds digit and sometimes with the tens digit.

Further Ideas

- Play the game using 2-, 4-, or 5-digit numbers according to the ability of the students.

Down to One

Junior high school

Objective

Develop a variety of mental methods of computation.
Develop the use of the four operations to solve problems.
Use sequence methods of computation when appropriate to a problem.
Estimate and approximate solutions to problems.

Explanation of the activity

Use the calculator to generate a 3-digit random number.
The aim is to get the calculator to display the number 1.
Players can use any of the numbers 1 – 9 together with any of the keys below:

[÷] , [−] , [×] , [÷] , [(] , [)] , [=]

You cannot put numbers together to make 2- or 3-digit numbers.
You can use each number only once.
The first player to get his/her calculator display to show 1 scores five points.
If after an agreed time limit no player has reached 1, the player who is closest scores two points.

While working on this activity, students should develop their skills of mental mathematics and their fluency with numerical calculations.

Using the calculator

Calculator functions used: Subtraction, division, last answer memory

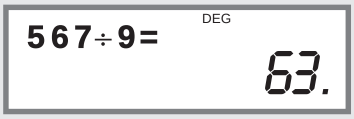
Press the following buttons and then start operation.

[ON/C] [2ndF] MODE [ ] [0]

Suppose the random number you generate is 567.

Example A:

[ON/C] 567 [÷] 9 [=]

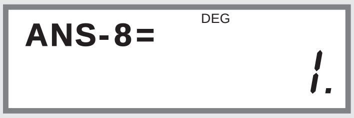


567÷9=
DEG
63.

[÷] 7 [=]

ANS÷7=
DEG
9.

[−] 8 [=]



ANS-8=
DEG
1.

The answer is 1 and the game is finished.

Down to One

Example B:

− 9 =

÷ 8 =

You want to subtract 8 from 9, but you cannot since you have already used 8 once.
So...

÷ 3 =

− 2 =

The

Studer should be
priate erations and
numb is divisible by 9,
This ll prompt students
thir about the various strat

Th game could be played betwee

Points

or some students, it may be more appropr
tor should be set to fixed decimal place mode
[.] key, which has FSE written in yellow above it,
screen. And press [2ndF] [TAB] and [0] keys. Doing
starting number can then be generated by multiplying a

Further Ideas

- Play the game using decimal starting numbers.
- Give the students a shuffled set of cards numbered from 1
  cards numbered 10, 20, 30, 40, 50. Students choose five cards
  two cards from the second set. The calculator is then used to ge
  digit integer, and the students have to make this total by using the n

Reverse the Order

Junior high school

Objective

Develop a variety of mental methods of computation.
Estimate and approximate solutions to problems.

Explanation of the activity

Enter any 2-digit number into the calculator.
Reverse the order of the digits through simple calculator operations.

While working on this activity, students should develop their skills of mental mathematics. They should also be interpreting and generalizing their answers.

Using the calculator

Calculator functions used: Addition, subtraction

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

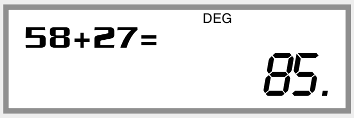
Example A:

To reverse the order of 58:

85 − 58 =

85-58=
DEG
27.

58 + 27 =



58+27=
DEG
85.

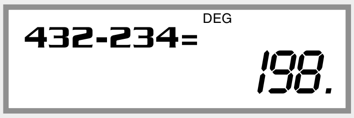
Solution: Add 27 to 58 to get 85.

Now try using a 3-digit number.

Example B:

Enter 432 into the calculator

ON/C 432 − 234 =



432-234=
DEG
198.

234 + 198 =

234+198=
DEG
432.

Solution: Add 198 to 234 to get 432.

Reverse the Order

Junior high school

Using the activity in the classroom

This activity is probably best introduced orally to a group of students. Ask the students to enter any two digit number into their calculators. Then, ask them to find a simple way to reverse the order of the digits of these numbers. Students may do this by using inverse operations.

Points for students to discuss

After trying an example, the students can talk about the operations and numbers that they used. This discussion should lead to the generalization that one way to reverse the order of the digits is to add or subtract a multiple of 9. More able students could be asked to try and prove this generalization:

$$(10a + b) + N = (10b + a)$$

$$N = (10b + a) - (10a + b)$$

$$N = 9b - 9a = 9(b - a)$$

Further Ideas

Try using the activity with 3-digit numbers, 4-digit numbers, etc.
Choose any 2-digit number, reverse it, and then add the reversed number to the original. What happens? Try this with 3-digit numbers or 4-digit numbers, etc.

Different Products

Junior high school

Objective

Estimate and approximate solutions to problems.

Explanation of the activity

Have the class make up multiplication problems using the digits 1, 2, 3 and 4. Each digit can only be used once. Find out what the largest product among the possible answers will be.

While working on this activity, students should practice their skills of mental estimation. They should also be interpreting and generalizing their answers.

$\boxed{\text{2ndF}}$

What is the largest number you can make by pressing the keys $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ and $\boxed{4}$ once and only once?

Example:

12 $\boxed{\times}$ 34 $\boxed{=}$

2 $\boxed{\times}$ 341 $\boxed{=}$

Can you make a larger number?
Using algebra, for any four digits a, b, c, d, where
$a < b < c < d$, the largest product is given by:
$(10d + a) \times (10c + b)$.

Ans: The largest product is given by

41 $\boxed{\times}$ 32 $\boxed{=}$

Different Products

Junior high school

Using the activity in the classroom

This activity could be introduced to the whole class by asking students to individually make up any multiplication using only the digits 1, 2, 3 and 4. The different multiplication problems and their answers can then be compared and students can be set the task of finding the largest product. Students should be encouraged to estimate the answers to the various multiplication problems.

Points for students to discuss

Students can explore other sets of four numbers, generalizing the rule to find the largest product using words or symbols. After generalizing, explain the rule that for any four digits a, b, c, d, where $a < b < c < d$, the largest product is given by:

$(10d + a) \times (10c + b)$.

If the investigation is extended to the five digits 1, 2, 3, 4, 5, then the largest product is given by:

$431 \times 52 = 22412$.

For some students it may be appropriate to begin with only three digits.

Further Ideas

- Find the largest product for any number of digits.
- Find the smallest product for any number of digits.
- Find the different sums that can be made by adding the digits 1, 2 and 3 once and only once. For example 12 + 3 = 15. What happens for other sets of 3-digit numbers?

Sums and Products

Junior high school

Objective

Calculate with decimals and understand the results.
Select suitable sequences of operations and methods of computation, including trial-and-improvement methods, to solve problems involving integers and decimals.

Explanation of the activity

Choose two numbers whose sum is 10.
Find out what the product of those two numbers would be.
Find the products of other pairs of numbers whose sum is 10.
Find out which number pair gives the largest possible product.

This activity helps to reinforce students' understanding of the mathematical terms 'sum' and 'product' and develops trial-and-improvement methods.

Using the calculator

Calculator functions used: Addition, multiplication, subtraction, parentheses

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Try to find the largest product of any two numbers whose sum is 10.

Example:

2 + 8 =

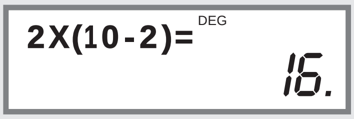

2+8= 10.

2 × 8 =


2X8= 16.

You can also calculate this as 2 x (10 - 2) = 16.

2 × (10 − 2) =


2X(10-2)= 16.

What two numbers give the largest product?
Try multiplying various combinations of numbers whose sum is 10.

Ans: 5 x 5 = 25

Sums and Products

Junior high school

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity could be introduced orally.
The largest product is 25, given by 5 x 5. Some students may need to be encouraged to consider decimal numbers to verify that the largest product is 25. More able students should be encouraged to try and prove that this is the largest product.
One method of using the calculator is to enter the product as two numbers that can be edited. Some students may prefer to enter the product as an expression such as 2 x (10 - 2), which can be edited.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

Students could be encouraged to devise similar problems to give to each other involving numbers with different sums.

Further Ideas

- Investigate products of 3, 4, 5... numbers which have the same sum. This could be explored graphically.
  (Generally, for two numbers whose sum is n, the largest product is given by $(n/2)^2$, for three numbers whose sum is n, the largest product is given by $(n/3)^3$... The nearest integer to (n/e) where $e = 2.718$ is the number of numbers which will give the maximum product.)

- The problem of finding two numbers whose product is a given total can be turned into a game where students score points according to the number of trials they perform to identify the solution. For example: The sum of two numbers is 10 and their product is 19.71. What are the two numbers?

  Ans: The two numbers whose product is 19.71 are 7.3 and 2.7.

Target 100

Junior high school

Objective

Understand and use the concept of place value in whole numbers and decimals, relating this to computation.
Calculate with decimals and understand the results; e.g. multiplying by numbers between 0 and 1.
Mentally estimate and approximate solutions to numerical calculations.

Explanation of the activity

A game for two players.
- Player 1 enters any 2-digit number into the calculator.
- Player 2 then multiplies this by another number so that the answer is as close as possible to 100.
- Players score points according to how close they are to 100:
  - within 10 = 1 point
  - within 5 = 2 points
  - within 1 = 5 points
  - exactly 100 = 10 points
- Player 2 then enters a number and the game continues.
- The first player to score 20 points wins.

While working on this activity, students will be extending their understanding of decimals and improving their estimation skills.

Using the calculator

Calculator functions used: Multiplication

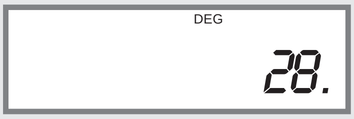
Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example:

Player 1 enters 28.

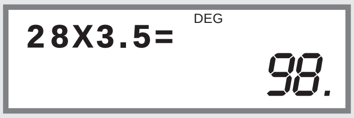
28


DEG

28.

Player 2 multiplies this by 3.5.

× 3.5 =


DEG

28X3.5=

98.

Player 2 scores two points.

The game continues until one player reaches 20 points.

Target 100

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity could be given to students with little introduction from the teacher. Alternatively, the game could initially be played between the teacher and a large group of students. It is important that students are encouraged to think carefully about the numbers they choose and that the teacher focuses on the students' mental skills. Most benefit is obtained from the activity when students are playing together in small teams, discussing their choices of a number to multiply by.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

At the end of the activity, students' strategies should be discussed and compared.

Further Ideas

- Play the game with different target numbers. For example, students could multiply or divide a random number to reach a target of 1.

- The first player multiplies a random number to aim for a target of 100. The second player then multiplies this answer to try and get even closer to 100. The player who gets the calculator to display a number between 99 and 101 wins.

Ordering Fractions

Junior high school/
Elementary school
(upper grades)

Objective

Understand and use fractions.
Understand the interrelationship between fractions and decimals.

Explanation of the activity

Estimate where a given fraction would be located on a numerical line.
Check the answer using the calculator.
While working on this activity, students will be developing their understanding of the relative sizes of common fractions. The activity suggests an approach to teaching equivalence of common fractions.

Using the calculator

Calculator functions used: Addition, division, fractional calculation

You will need a 0 – 2 number line.
Estimate where the following common fractions should be placed on the number line and then record estimates.

$\frac{1}{2}, \frac{15}{8}, 1\frac{2}{5}, \frac{4}{6}, \frac{3}{4}, \frac{4}{3}, \frac{14}{10}, \frac{2}{3}, 1\frac{5}{6}, \frac{3}{5}$

Use a calculator and a ruler to check your estimates.

Example A:

Find the value of the fraction $\frac{1}{2}$.
Using division:

1 $\boxed{\div}$ 2 $\boxed{=}$

Using fractional calculation:
1 ⌐ 2 on the calculator display means $\frac{1}{2}$.

$\boxed{\text{ON/C}}$ 1 $\boxed{a\,b/c}$ 2 $\boxed{=}$

Convert $\frac{1}{2}$ to decimal notation.

$\boxed{a\,b/c}$

Ordering Fractions

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity may be introduced orally. The number line could be copied onto an overhead projector transparency or written on the board. Divide students into small groups and give each group a fraction card. Have the groups discuss where to place their given fraction on the line. Groups then take turns marking their fractions on the number line. Solutions can be discussed, together with methods of checking the solutions. This should lead into converting common fractions to decimal notation, and students can be shown how to do this on the calculator. It is important that students are aware of the general method of converting common fractions into decimal notation (dividing the numerator by the denominator), as well as the use of the fraction key on the calculator.

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It will be discovered that some of the fractions are equivalent to each other and this leads into the second part of the activity. When the fraction 4/6 is entered into the calculator, pressing '=' simplifies the fraction to 2/3. Students should explore the results of entering different fractions, thus generating sets of equivalent fractions. It is important that students are encouraged to understand the concept of equivalence.

Further Ideas

Small groups of students are given a pack of cards with a different fraction on each card. The

Adding Fractions

**Junior high school/
Elementary school
(upper grades)**

Objective

Understand and use fractions.
Calculate with fractions and understand the results.

Explanation of the activity

Using the calculator, find the sum of two given fractions each having 1 in the numerator.
Look for patterns to help understand how to add the fractions without using the calculator.
This activity suggests an approach to teaching addition of common fractions.

Using the calculator

Calculator functions used: Addition, fractional calculation

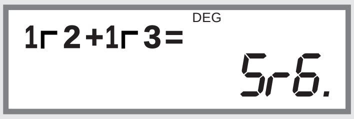
Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example:

Using fractional calculation, find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

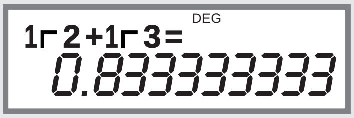
1 ab/c 2 + 1 ab/c 3 =


1r2+1r3=
5r6.

$\frac{1}{2} + \frac{1}{3} =$ on the calculator display means $\frac{5}{6}$.

Convert $\frac{5}{6}$ to decimal notation.

ab/c


1r2+1r3=
0.833333333

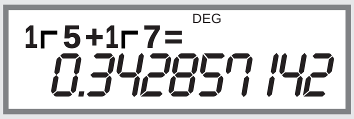
0.83333··· ab/c on the calculator display means $\frac{5}{6}$.

Find the sums of other common fractions.

1 ab/c 5 + 1 ab/c 7 =


1r5+1r7=
12r35.

ab/c


1r5+1r7=
0.342857142

Adding Fractions

Junior high school/
Elementary school
(upper grades)

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity should be presented after studying equivalence of common fractions.

The activity is best introduced orally. Ensure that the students know how to add two common fractions on the calculator. Ask them to add 1/2 and 1/3 and record the answer (5/6). Ask the students if they can see any connection between the answer and the original two fractions. Students may note that 2 + 3 = 5 and 2 x 3 = 6. Allow students to explore other unit fractions and encourage them to generalize. Students should be asked to try and explain what is happening. It should be noted that the pattern may appear to break down when fractions with a common denominator are added.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

Students can then explore what happens when other common fractions are added. For some students, it may be appropriate to begin by considering a pair of fractions that includes one unit fraction.

It is important that students are encouraged to understand what is happening, and that reference is made to equivalent fractions.

Further Ideas

- Investigate subtracting, multiplying or dividing common fractions.

- The Babylonians mostly used fractions which had 1 as the numerator. For example, 5/6 could be written as 1/2 + 1/3. Investigate Babylonian fractions.

Halfway Between

Junior high school/
Elementary school
(upper grades)

Objective

Understand and use fractions.
Calculate with fractions and understand the results.

Explanation of the activity

Use the calculator to find the fraction that is exactly halfway between two other fractions.
Look for patterns to help understand how to find the answer without using the calculator.

This activity reinforces addition of common fractions and considers the result of dividing common fractions by integers. By working on the activity, students should also develop an increasing feel for the relative sizes of fractions.

Using the calculator

Calculator functions used: Addition, division, multiplication, fraction, calculation

Press the following buttons and then start operation.

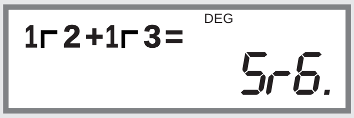
$\boxed{\text{ON/C}}$ $\boxed{\text{2ndF}}$ $\overset{\text{MODE}}{\boxed{}}$ $\boxed{0}$

Example A:

Find the fraction that is halfway between $\frac{1}{2}$ and $\frac{1}{3}$.

Using fractional calculation, obtain the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

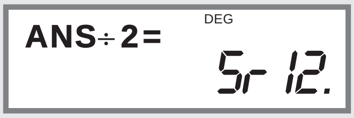
1 $\boxed{a\,b/c}$ 2 $\boxed{+}$ 1 $\boxed{a\,b/c}$ 3 $\boxed{=}$


1⌐2+1⌐3=
5⌐6.

<Display 1>

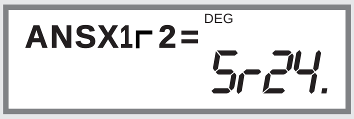
Half of this fraction is the number you are looking for, so divide this fraction by 2.

$\boxed{\div}$ 2 $\boxed{=}$


ANS÷2=
5⌐12.

Or after <Display 1>, multiply by $\frac{1}{2}$.

$\boxed{\times}$ 1 $\boxed{a\,b/c}$ 2 $\boxed{=}$

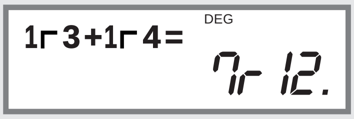

ANSX1⌐2=
5⌐24.

Example B:

Find the fraction that is halfway between $\frac{1}{3}$ and $\frac{1}{4}$.

Using fractional calculation, obtain the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

1 $\boxed{a\,b/c}$ 3 $\boxed{+}$ 1 $\boxed{a\,b/c}$ 4 $\boxed{=}$


1⌐3+1⌐4=
7⌐12.

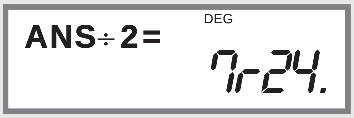
<Display 2>

Halfway Between

Junior high school/
Elementary school
(upper grades)

Half of this fraction is the number you are looking for, so divide this fraction by 2.

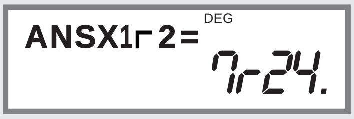
÷ 2 =



ANS÷2=
DEG
7⌐24.

Or after <Display 2>, multiply by $\frac{1}{2}$.

× 1 a b/c 2 =



ANSX1⌐2=
DEG
7⌐24.

Continue the activity using other common fractions.

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity could follow the study of addition of common fractions.
The activity is best introduced orally. Ask the students to give different fractions that lie between 5/12 and 2/3. One possibility is to arrange these on a fraction line. It is important that students are challenged to justify their answers and, in some cases, it may be appropriate to consider decimal equivalents. The students should then be asked to identify the common fraction that is halfway between 5/12 and 8/12, justifying their answer.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

Furthering the activity, students can be asked to give fractions that lie between 1/2 and 1/3 and identify the common fraction that is halfway between them. At this stage it may be necessary to discuss methods for finding a number that is halfway between two numbers. Students can then use their calculators to identify fractions that are halfway between other unit fractions. This can be extended to non-unit fractions. It is important that students are encouraged to understand what is happening.

Further Ideas

- Find fractions that lie 1/3 of the way between two fractions, or 1/4 of the way between two fractions, etc.

Near Integers

Junior high school

Objective

Understand and use the concept of place value in decimals.
Understand and use decimals and fractions while comprehending the interrelationship between them.
Use some common properties of numbers, including multiples.
Give solutions in the context of the problem, selecting the appropriate degree of accuracy and interpreting the display on a calculator.

Explanation of the activity

Use the calculator to find fractions that are near integers in decimal form.

While working on this activity, students will be developing their understanding of decimals, particularly their relationship with fractions.

Using the calculator

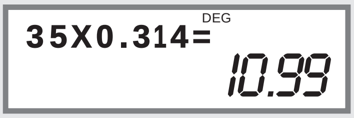
Calculator functions used: Multiplication, division

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

The 35th multiple of 0.314 is 10.99.
35 x 0.314 = 10.99

35 × 0.314 =

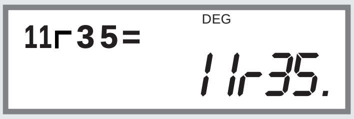


DEG
35X0.314=
10.99

10.99 is a ‘near integer’; it is nearly 11.

Using fractional calculation, input $\frac{11}{35}$.

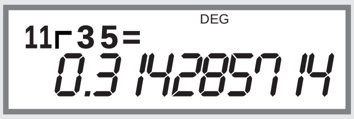
ON/C 11 a b/c 35 =



DEG
11┌35=
11┌35.

Convert $\frac{11}{35}$ to decimal notation.

a b/c

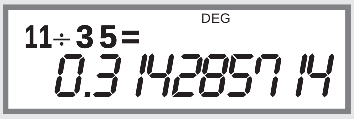


DEG
11┌35=
0.314285714

Using division, divide 11 by 35.

The fraction $\frac{11}{35}$ has a decimal value close to 0.314.

11 ÷ 35 =



DEG
11÷35=
0.314285714

Near Integers

Junior high school

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity is probably best introduced orally. Students could use the sequence function of the calculator to generate the multiples of some integers, and could then begin to investigate the multiples of some decimals.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

The teacher could ask the students to generate the multiples of 0.314, challenging them to find a multiple that is nearly an integer. Students can then begin to investigate the situation further.

Further Ideas

- Use this idea to investigate different approximations for π. For example, 22/7 = 3.142857, whereas $\pi \approx 3.141593$. However, 179/57 = 3.140351.

- Investigate approximations for $\sqrt{2}$, or $\sqrt{3}$, etc.

Reshaping Cuboids

Junior high school

Objective

Reinforce students' understanding of the equivalence of shapes in various alignments and how this relates to multiplication within a practical context.
Develop mental skills involving factors, divisors, and systematic thinking.

Explanation of the activity

12 cubes, each with a volume of 1 cm^3, may be placed together to create any of four cuboids, each having a volume of 12 cm^3.

Find the equivalent equations for each of the cuboids; for example,
1 x 1 x 12 = 12, 2 x 2 x 3 = 12, etc.

Using the calculator

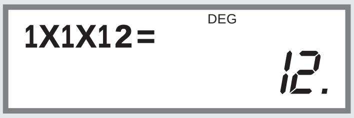
Calculator functions used: Multiplication, Multi-line Playback

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Introduce students to the calculator's Multi-line playback feature, which will be useful to display sets of solutions for each "volume" number.

1 × 1 × 12 =



1X1X12= DEG 12.

2 × 2 × 3 =

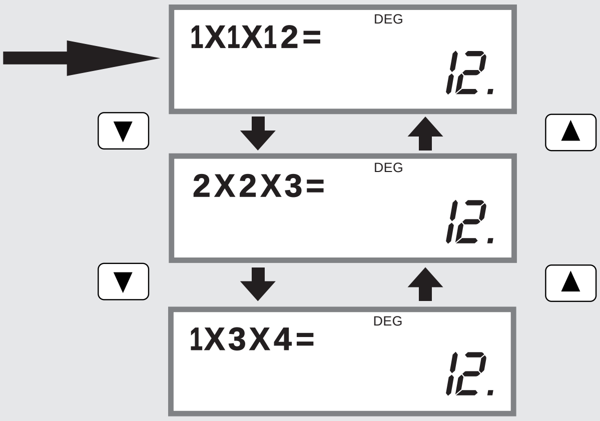
2X2X3= DEG 12.

1 × 3 × 4 = etc.

1X3X4= DEG 12.

Each press of a ▲ ▼ key takes you one calculation step forward or backward.

2ndF ▲



Display the first calcualtion with 2ndF ▲

Reshaping Cuboids

Junior high school

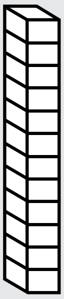
Find the five calculations that represent cuboids that each have a volume of 30 cm^3.

e.g. 1 $\boxed{\times}$ 1 $\boxed{\times}$ 30 $\boxed{=}$ etc.

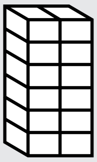
In a similar way, find the twelve calculations for cuboids each having a volume of 96 cm^3.

How many similar calculations must there be for 180 cm^3?
Which of these cuboids is nearest to looking like a cube?

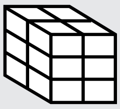
For volumes between 150 cm^3 and 200 cm^3, which particular ones can be represented by at least 16 cuboids each? Which volumes have the smallest number of cuboids each?



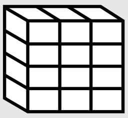
1 x 1 x 12 = 12



1 x 2 x 6 = 12



2 x 2 x 3 = 12



1 x 3 x 4 = 12

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

Students may benefit from the use of actual blocks that can be stacked to form the different cubic combinations. An OHP calculator could also be used to collect solutions from the entire class.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

The number of divisors for a number expressed as p^a x q^b x r^c (where p, q, and r are all prime) is (a + 1) (b + 1) (c +1). For example, 360 = 2^3 x 3^2 x 5^1. Here, a = 3, b = 2, and c = 1, so the number of divisors is given by the expression (3 + 1) (2 + 1) (1 + 1) = 24. Therefore, 360 has 24 divisors.

Further Ideas

- Use trial and improvement to find the side of a cube having a volume of 180 cm^3.

- Move into "four (or more) dimensions" as a means of finding the factors of a number. For example, 6006 = 77 x 78 = (7 x 11) x (6 x 13) = 2 x 3 x 7 x 11 x 13. All stages can be displayed using the replay function.

Function Tables

Junior high school

Objective

Understand and use calculator functions.
Understand and apply functional relationships.
Enable speedy plotting of graphs.

Explanation of the activity

Use the calculator to calculate the *y* values for a given function using a set range of values for *x*. Record the values on a table and use them to plot a graph.

Using the calculator

Calculator functions used: Multiplication, editing, Multi-line Playback

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

x	-5	-4	-3	-2	-1	...	5
y	-25	-21	-17	-13	-9	...	15

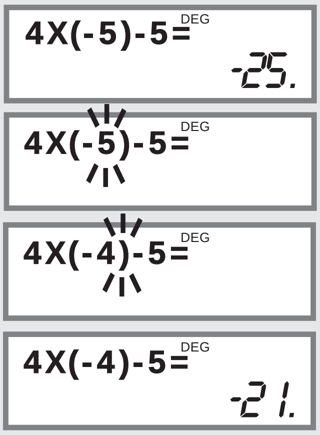
Enter the *y* values for the function *y* = 4*x* - 5 using the values from -5 to +5 for *x*. Use the calculator's playback function to calculate the functions efficiently. After calculating the values, use them to plot the graph of *y* = 4*x* - 5.

4 × 5 +/- − 5 =

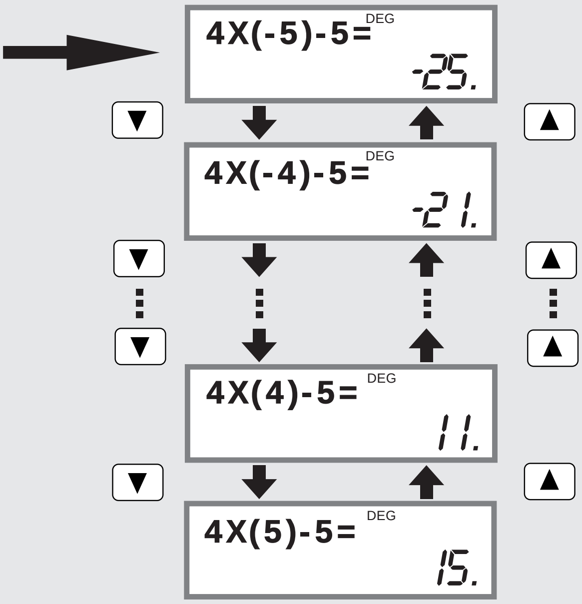
▶ ▶ ▶ ▶ ▶

4 DEL

=



2ndF ▲



In the same way, find the values for *x* = -3, -2, 1, 0,...5.

Each press of a ▲ ▼ key takes you one calculation step forward or backward.

Display the first calcualtion with 2ndF ▲

Return to the calculation for $x = 5$ and redo the calculation for the equation $y = 4x - 3$.
Add another line to the table and calculate the values of y for the new equation.
Plot the second graph with the first on the same axis.

What do you notice about the graphs and the numbers in the table?

What do you think will happen if you try another similar equation such as $y = 4x - 1$, $y = 4x + 1$, or $y = 4x + 4$?

Can you explain the number pattern and the picture you have produced?

Using the activity in the classroom

This activity should be introduced after practicing substitution.

Start the activity as a whole class so the students can gain confidence in using the calculator and see the advantages of calculating first and then recording the results to speed up the process of making the graph table. The students can calculate the y values for the second equation themselves and quickly continue with other suggested equations using multi-line playback to go directly from the (x, y) values to the graph without needing to record the result in a table. This enables the families of graphs to be compared rapidly. Try extending the activity by using graphs with different gradients to establish the parallel nature of the graphs, and then try keeping the intercept constant and varying the gradient.

Points for students to discuss

The idea of using the playback function as a rapid way to calculate function values can be applied to a wide range of equations including polynomials, trigonometric functions, etc. Students can do calculations in one sequence and then use the playback function to go back through the answers and record or plot them all at once.

Further Ideas

Investigations on graphs can be done more quickly if the playback function is used so each function does not have to be retyped at every entry. Demonstrate this by using the following suggestions:

- Solve a quadratic function such as $ax^2 + bx + c = 0$ for varying values of a, b, and c.

- Use the calculator to generate values of a trigonometric function and enter the results directly onto a graph using the playback function.

Palindromes

Junior high school

Objective

Understand and use the concept of place value in whole numbers.
Explore a variety of situations that lead to the expression of relationships.
Construct and interpret formulas and expressions.
Manipulate algebraic expressions.

Explanation of the activity

A word that reads the same forwards and backwards, such as “mom” and “level”, is called a palindrome. A palindromic number is exactly the same; the number has the same value whichever way you write the digits. For Example, 212, 34543 and 10001.

Using the calculator

Calculator functions used: Division

Press the following buttons and then start operation.

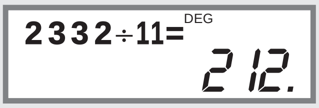
ON/C 2ndF MODE 0

Write down some 4-digit palindromes and use the calculator to divide each of them by 11.

Example:

ON/C 2332 ÷ 11 =

5665 ÷ 11 = etc.


2332÷11=
DEG
212.


5665÷K=
DEG
515.

Do you notice a pattern?

The use of the playback function will speed up the calculations and enable students to compare results to look for a pattern.

Make up some more 4-digit palindromes and divide each of them by 11.
Have students compare their results until they notice a pattern.

Palindromes

Junior high school

Using the activity in the classroom

This is an activity that can first be given to students to work on and the results later discussed as a group. The object is to discover a pattern in the results—the pattern being that the answer will always be a whole number.

Points for students to discuss

The pattern may be too obvious for the students to mention, so it may be necessary to give them the following hint:
Try writing the number in its long form; for example,

$$2332 = 2 \times 1000 + 3 \times 100 + 3 \times 10 + 2$$

Further Ideas

- Try to explain the problem using algebra. You could start off by giving the class just the first line of the calculation below and let them work on the rest individually or in groups.

$$\begin{aligned} a\,b\,b\,a &= a \times 1000 + b \times 100 + b \times 10 + a \\ &= a \times 1000 + a + b \times 100 + b \times 10 \\ &= a\,(1000 + 1) + b\,(100 + 10) \\ &= a \times 1001 + b \times 110 \\ &= a\ 91 \times 11 + b \times 10 \times 11 \\ &= 11\,(91a + 10b) \end{aligned}$$

- Try the activity using 6-digit palindromes.
  Have the class prove that not all 5-digit palindromes are exactly divisible by 11.

Trial and Improvement

Junior high school

Objective

Mentally estimate and approximate solutions to numerical calculations.
Understand and use the concept of place value in decimals and relate it to computation.

Explanation of the activity

Use “trial and improvement” to find the length of the side of a cube-shaped box that can hold 100 cm^3 of ice cream.
The two mental calculations 4 x 4 x 4 = 64 and 5 x 5 x 5 = 125 should suggest a possible starting calculation such as 4.5 x 4.5 x 4.5 = 91, which can be shortened to 4.5^3 = 91.
This activity gives students the opportunity to enhance their understanding of decimals and improve their skills in estimation.

Using the calculator

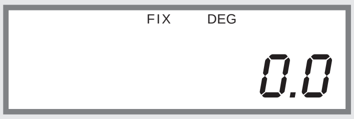
Calculator functions used: Multiplication, FSE, TAB

Press the following buttons and then start operation.

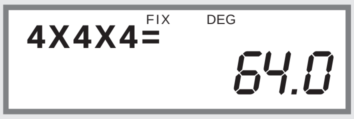
ON/C 2ndF MODE 0

Set the calculator to “fixed point” notation with a TAB value of 0.
(Doing this will display answers to the nearest whole number.)
Adjust the TAB setting to 1 and then continue to improve the accuracy of the answer

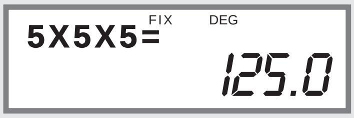
2ndF FSE 2ndF TAB 1



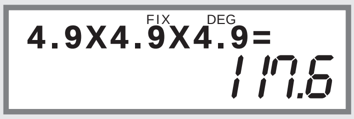
4 × 4 × 4 =



5 × 5 × 5 =

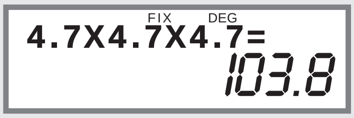


4.9 × 4.9 × 4.9 =

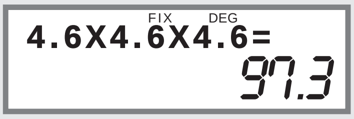


⋮

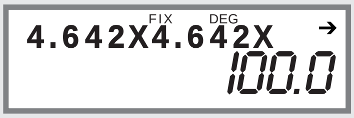
4.7 × 4.7 × 4.7 =



4.6 × 4.6 × 4.6 =



From this we can see the answer lies between 4.6 and 4.7. Continue to search for the answer repeating this operation.



Switch FSE and TAB to normal display for further operation.

ON/C 2ndF FSE 2ndF FSE 2ndF FSE

Press FSE until FIX, SCI, or ENG are not shown on the display.

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity may be given to students with little introduction or, with the use of the OHP unit, this or a similar task may be introduced to the whole class followed by individual work on one or more of the extension activities. The use of the multi-line playback function will be of practical benefit in tackling questions involving trial and improvement.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

It will be necessary to familiarize the students with the FSE and TAB keys in order to understand, for example, why 4.641^3 and 4.642^3 both have the value 100 to the nearest unit. In the context of similar problems, students will need to consider what degrees of accuracy are appropriate; in the case of cubic centimeters of ice cream, possibly only to one decimal place.

Further Ideas

- Find the side of a cubical carton whose volume is 1/2 liter. It may be necessary to remind students of the equivalence of 500 ml (fluid measure) and 500 cm^3 (solid measure).

- Find the dimensions of a fruit juice carton whose sides are in the proportion 1 : 2 : 3 and whose capacity is 1 liter.

- Find the Golden Ratio x by trial and improvement of the relation

  Guess x (Guess + 1) = 1

Use the playback function on the calculator to show that

$x = 1 / (1 + x)$ and that $x = \sqrt{(1 - x)}$.

All metric paper has the same shape (except golden). If A0 has an area of 1 m^2 and the longer side is $\sqrt{2}$ times bigger than the smaller side, find these dimensions. What are the dimensions of A4? Have the students confirm their calculations by measuring a sheet.

Last Digits

Junior high school

Objective

Use last digits as a means of checking the output of a calculator.
Practice estimation and observe patterns.
Reinforce the concept of prime numbers.

Explanation of the activity

Perform a series of multiplication equations keeping the last digit of each of the multipliers constant; for example, 3 x 7, 13 x 7, 3 x 17, etc.

Using the calculator

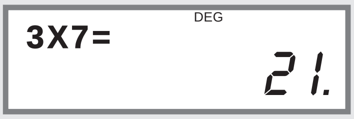
Calculator functions used: Multiplication

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Enter the following equations into the calculator:

3 × 7 =



```
3X7=
21.
```

13 × 7 =

```
13X7=
91.
```

3 × 17 =

```
3X17=
51.
```

Find other last digit combinations that give answers ending in 1.

Which of the numbers in the following set can be made from the product of two numbers? (excluding equations using 1 multiplied by the number itself)

21, 41, 51, 61, 71, 81, 91, 101, 111, 121, 131, 141, 151, 161, 171, 181, 191, 201

Which of the numbers can be made in more than one way?

Make a collection of your calculations so that they can be displayed in order of answer size.

Name the type of numbers that cannot be made.

Last Digits

Junior high school

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

It is probably best to first introduce the activity as a class to give the students an opportunity to make estimates before using their calculators. Once the class has shared their initial ideas, they can be given time to investigate any patterns they discover.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

After investigating patterns on their own, students should share their discoveries with the rest of the class.

Further Ideas

- Examine the first 20 prime numbers. Except for the number 2, they all end with an odd last digit. Repeat the procedure for the last digits of 3, 7, and 9. Find all the prime numbers between 1 and 201.

- Find the last digits to:
    1. the answers to the multiplication tables.
    2. the square numbers.
    3. other number sequences such as the cube numbers and triangle numbers.

- Find the two consecutive numbers whose product is 6006.

A Question of Interest

Junior high school

Objective

Understand, use and calculate with percentages.
Select suitable sequences of operations and methods of computation, including trial-and-improvement methods, to solve problems involving integers, decimals and percentages.
Give solutions in the context of the problem, selecting an appropriate degree of accuracy, and interpret the display on a calculator.

Explanation of the activity

Use the calculator to find solutions to problems involving interest rates.
While working on this activity, students will develop their understanding of percentages within the context of compound interest situations.

Using the calculator

Calculator functions used: % calculation, multiplication

Press the following buttons and then start operation.

[ON/C] [2ndF] [MODE] [0]

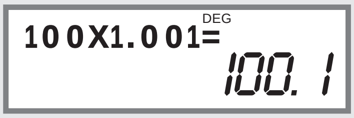
If you invest money at a certain level of interest, by how much will your money grow?

Example:

$100 is invested at 0.1% annual interest.

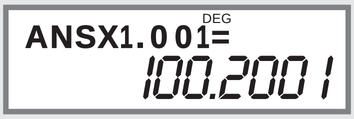
Using multiplication: Multiply the principal $100 by 1.001.

100 [×] 1.001 [=]


DEG
100X1.001=
100.1

For the total after two years, multiply the previous answer again by 1.001.

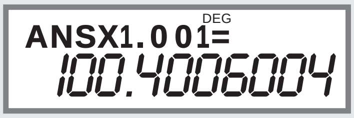
[×] 1.001 [=]


DEG
ANSX1.001=
100.2001

After three years... [=]

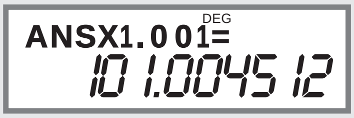

DEG
ANSX1.001=
100.3003001

After four years... [=]


DEG
ANSX1.001=
100.4006004

After 10 years, you have approximately $101.

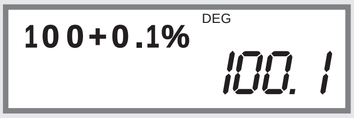
[=] [=] [=] [=] [=] [=]


DEG
ANSX1.001=
101.004512

A Question of Interest

Using the % calculation key: After one year, you should have 0.1% of your $100.

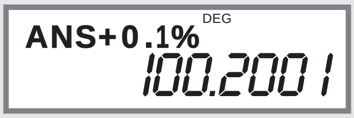
100 [+] 0.1 [2ndF] [%]



DEG
100+0.1%
100.1

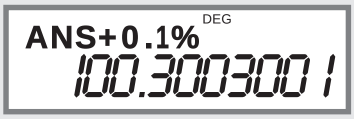
You now have $100.10.

After two years, you have 0.1% more. [+] 0.1 [2ndF] [%]



DEG
ANS+0.1%
100.2001

After three years... [2ndF] [%]

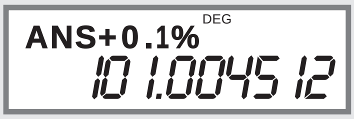


DEG
ANS+0.1%
100.3003001

After 10 years...

[2ndF] [%] [2ndF] [%] [2ndF] [%] [2ndF] [%] [2ndF] [%] [2ndF] [%] [2ndF] [%]

You have approximately $101.



DEG
ANS+0.1%
101.004512

Using the activity in the classroom

This activity is probably best introduced orally. After a discussion about investments and interest rates, the teacher can use the sequence function of the calculator to generate sequences showing how an initial capital sum grows for a fixed interest rate. Students can be asked to find the annual interest rate that ensures their money is doubled in 10 years.

Students can then investigate the annual interest rates that would double their money for different numbers of years. These interest rates could be plotted on a graph.

5 years	14.9%
10 years	7.2%
15 years	4.7%
20 years	3.6%
25 years	2.8%

Points for students to discuss

It may be useful to show students how to generate sequences on the calculator.

Further Ideas

- Investigate interest rates that would triple an investment, or...
- From 1970 to 1980 prices tripled. What was the average rate of inflation?

For High school Students

How much will your investment be worth in *n* years?
Let’s make an equation.

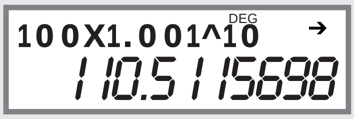
The original amount of money invested, called the principal, multiplies each year by the amount *x*.
Let’s use this equation to see how much money we have after 100 years.

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

The original amount, or principal, is $100; so $a = 100$.
The number of years is 100; so $n = 100$.
The interest is 0.1%; so $x = 1.001$.

100 × 1.001 y^x 100 =



DEG
100X1.001^10 →
110.5115698

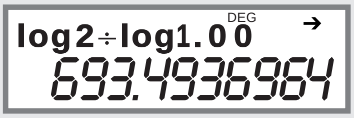
You have $110.50 after 100 years.

How many years would it take for the money to double?
Let’s make an equation.
The money invested multiplies each year by the amount *x*.
After *n* years the money doubles, so...

$ax^n = 2a$
Divide both sides by ‘*a*’ $x^n = 2$
Calculates the ‘log’ of both sides $\log x^n = \log 2$
$n\log x = \log 2$
$\therefore n = \log 2 / \log x$

If ‘*a*’ is the money deposited, the savings would double.

log 2 ÷ log 1.001 =



DEG
log2÷log1.00 →
693.4936964

It takes approximately 694 years for your money to double.

Getting Even

Elementary school (upper grade)

Objective

Use some common properties of numbers.
Explore a variety of situations that lead to the expression of relationships.

Explanation of the activity

A game of chance to compare the relationship between odd and even numbers.
By working on this activity, students will reinforce their understanding of odd and even numbers. More able students could develop their skills in using algebra to prove generalizations.

Using the calculator

Calculator functions used: Addition

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

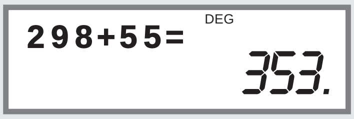
A game for two players

The first player enters any number into his/her calculator without showing it to the other player.
For example, 298.

The second player then enters a number into his/her calculator without showing it to the other player.
For example, 55.

The players then show each other their numbers and add them. If the answer is even the first player scores 1 point; if the answer is odd, the second player scores 1 point.

ON/C 298 + 55 =



The first player to score 10 points is the winner.

Getting Even

Elementary school
(upper grade)

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

The game is best played between pairs or small groups of students. It could be introduced by the teacher playing the game against some students.

While playing the game, students should be encouraged to reflect on whether the game is fair, and also try and think about the reasons for their conjectures.

Odd + Even = Odd
Even + Odd = Odd
Odd + Odd = Even
Even + Even = Even

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

More able students could try to formally prove their conjectures.

The idea can be extended by students thinking about the conditions for obtaining even or odd answers when three numbers are added, or four numbers, or...

Further Ideas

More able students could try to formally prove their conjectures.

The idea can be extended by students thinking about the conditions for obtaining even or odd answers when three numbers are added, or four numbers, or...

Generating Sequences

Junior high school

Objective

Appreciate the use of letters to represent variables.
Explore number patterns arising from a variety of situations.
Interpret, generalize and use simple relationships, and generate rules for number sequences.

Explanation of the activity

Express simple functions initially in words and then symbolically.
Initially this will be in words, using 'ANS' to represent the variable, but this should lead to using the letter '*a*'.
Through discussion, it is possible to develop understanding of algebraic equivalence; for example '*a* + *a*' is equivalent to '2 x *a*', which is often written as '2*a*'. This can lead to simplifying algebraic expressions.
While working on this activity, students will be developing skills of generalization and refining their methods of expressing mathematical rules.

Using the calculator

Calculator functions used: Addition, subtraction, multiplication, division,
last answer memory

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Let's look at a sequence, find a rule to generalize it, and check it with our calculators.

Example:

3, 5, 7, 9, 11…

The above sequence can be described as an 'ANS + 2' sequence. This can be confirmed by adding 2 to 3, then 2 to this sum, and so on.

Add 3 and 2.

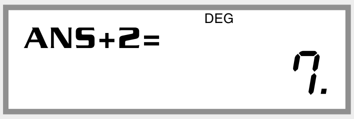
3 + 2 =

DEG
3+2=
5.

Compare the result of your calculation with each number in the sequence.

Add 2 to this sum.

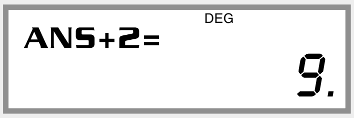
+ 2 =



DEG
ANS+2=
7.

Again, add 2 to this sum.

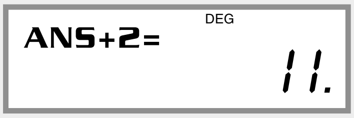
(+ 2) =



DEG
ANS+2=
9.

And so on.

=



DEG
ANS+2=
11.

We were thus able to confirm our 'ANS + 2' rule.

Try making rules to describe other number sequences.

Generating Sequences

Junior high school

• • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • •

This activity could be introduced orally. Read the terms of a sequence one at a time, asking students to identify the rule you are using. Repeat for other sequences and discuss the different rules. Below are some possible solutions. It is important to emphasize that there may be different solutions.

ANS + 4
ANS - 5
2 ANS
ANS ÷ 3
ANS x 3 - 1
$\text{ANS}^2 + 1$

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It may be useful to show students how to generate sequences on the calculator.

Further Ideas

Find different sequences that begin 1, 2, 5...
Is it possible to generate the multiples of any number, square number, triangle number?

Train Journeys

Junior high school

Objective

Develop and understand the relationship between units.
Understand and use compound measures including speed.
Undertake purposeful inquiries based on data analysis.

Explanation of the activity

This activity provides students with the opportunity to handle real data and work on a wide range of activities involving time, distance and speed.

Using the calculator

Calculator functions used: Degrees/minutes/seconds key (D°M'S),
time calculation (substraction, division)

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

The information on the table below shows the departure and arrival times of two trains, together with the distance of each station from Norwich.
How long does it take to complete the various stages of the journey?

Norwich – Diss	17.75
Diss – Stowmarket	13.5
Stowmarket – Ipswich	10.25
Ipswich – Colchester	15.25
Colchester – Chelmsford	19.5
Chelmsford – London	26

Example:

Try calculating the average speed of a train traveling from Norwich to London. The train leaves Norwich at 11:30 and arrives in London at 13:22. Therefore, the time for this journey is calculated by subtracting 11:30 from 13:22.

Input 13:22.

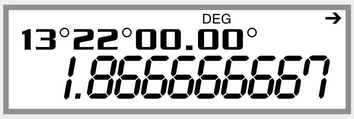
13 D°M'S 22

Subtract 11:30

− 11 D°M'S 30 =

We can see the journey takes one hour and 52 minutes.
How many hours is this?
Let's find out.

2ndF ↔DEG



One hour and 52 minutes is 1.8666 hours.

Train Journeys

Junior high school

Let's find the average speed at which the train travels between Norwich and London.

Speed = distance ÷ time
115 miles ÷ 1:52 is calculated as:

115 $\boxed{\div}$ 1 $\boxed{\text{D°M'S}}$ 52 $\boxed{=}$

```
115÷1°52°00.
     61.60714286
```

The train travels at approximately 61.6 mph.

How long does it take to complete the various stages of the journeys?

Why is the train that starts at Colchester described as a slow train?

What conjectures can you make about the different stages of the journeys?

• • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • •

This activity could be used with small groups, with each group looking at different aspects of the situation. For example, mean distances, times and speeds and relationships between distance and time. Students could investigate the average speeds for various stages of the journey, considering reasons for the differences in average speed.

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

Students could use the degrees/minutes/seconds key (D°M'S) to enter and compute various times. The actual purpose of the key should be discussed with students, however.

Further Ideas

- Compare other train lines and other forms of transport.
  What factors affect the speed of a train?

- Investigate the statement: "Longer journeys are quicker."

Simulated Dice

Junior high school

Objective

Use computers as a source of large samples and as a means to simulate events.
Understand and use relative frequency as an estimate of probability, and judge when sufficient trials have been carried out.
Recognize situations where probabilities can be based on equally likely outcomes.

Explanation of the activity

Use the calculators to simulate rolling dice.

While working on this activity, students will be developing a feel for probability. In particular, the relationship between estimating probabilities from experimental evidence and calculating probabilities based on equally likely outcomes.

Using the calculator

Calculator functions used: Random numbers, addition, fixed number calculation

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

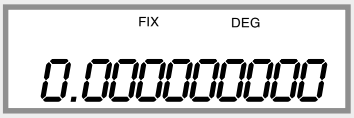
Let's use our calculators to simulate rolling dice.

Example:

To simulate rolling dice, we want the numbers from 1 to 6, so the formula will be
1 + RANDOM(5).

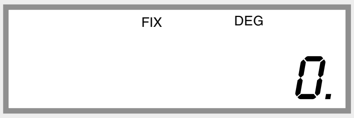
However, this will also generate decimals. So, change to the FIX mode by pressing TAB and 0 until appears FIX on a screen. Now only integers from 1 to 6 will be generated.

2ndF FSE



Set TAB to zero so that values to the right of the decimal point will not be displayed. (Values to the right of the decimal point are rounded to the nearest whole number.)

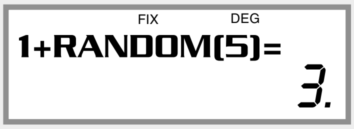
2ndF TAB 0



To generate random numbers from 1 to 6, calculate the formula 1 + RANDOM(5).

Try this numerous times.

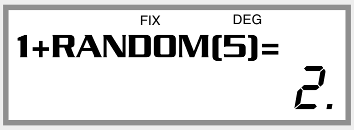
1 + 2ndF RANDOM (5) =



*1

Let's use the calculator to simulate the results obtained by adding the scores on two 1 – 6 dice.

=



*1

⋮

*1 It is the nature of random numbers that the value may not be the same for each time.

Simulated Dice

High school

Let’s create a simulation that throws these dice and finds their sum.
[1+RANDOM (5)] + [1+RANDOM (5)]
How good a simulation is this? Investigate other dice dimulations…

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity could form the basis of a whole class investigation. After some initial work on probability, students could be introduced to the idea of using the calculator to simulate dice.
The question can then be raised about the reliability of the simulation. The calculator can be used to simulate a die and the results can be collected, presented and analyzed. If the whole class works on the problem, a large number of trials can be carried out quite quickly. The results can be compared to expected frequencies based on equally likely outcomes.
Other ways of using the calculator to simulate dice can also be explored. For example, 0.5 + RANDOM (6). Students could then explore the reliability of other dice simulations, such as adding or multiplying the scores on two dice.

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It is probably best to explain orally to students how to use the calculator to simulate dice before starting the activity.

Further Ideas

- Compare the calculator’s simulation of dice to a computer’s simulation of dice.

After finishing this example, it is advised to return the display mode back to normal.

2ndF FSE

(Repeat until the FIX, SCI, ENG symbols are disappeared)

Mean Dice Scores

Junior high school

Objective

Use computers as a source of large samples and as a means to simulate events.
Calculate, estimate and use appropriate measures of central tendency with discrete data.

Explanation of the activity

Use the calculator to find out the mean score of consecutive rolls of a single die.

While working on this activity, students' understanding of the mean are reinforced. The activity also develops students' understanding of the concept of a limit, within the context of probability, and emphasizes the importance of experimentation and simulation.

Using the calculator

Calculator functions used: Random numbers, addition

Press the following buttons and then start operation.

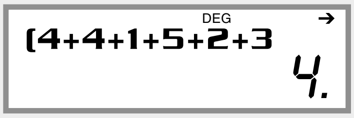
ON/C 2ndF MODE 0

Example 1:

Let's use our calculators to find out the mean score if we roll a die 10 times.

The mean is calculated by dividing the total score by the number of rolls of the die.
Calculate (4 + 4 + 1 + 5 + 2 + 3 + 6 + 5 + 4 + 6) ÷ 10.

(4 + 4 + 1 + ... + 6) ÷ 10 =


DEG
(4+4+1+5+2+3
4.

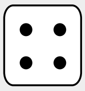
Let's see what happens to the mean score if we roll the die 20 times, 50 times, or 100 times.
What do you think will happen to the mean score as we continue to roll the dice?

Example 2:

Enter the data using statistics mode.

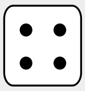
2ndF MODE 1


DEG
Stat x
0.



4 DATA

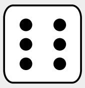

DEG
n=
1.



4 DATA


DEG
n=
2.

⋮



6 DATA


DEG
n=
10.

RCL x̄


DEG
$\bar{x}$=
4.
*

*You may not get the same results because random numbers are used.

Mean Dice Scores

High school

• • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • •

This activity could be introduced to a group using a real die. After some practical work, the need to roll the die many times suggests that the data may best be generated by computer simulation. The mean score for one die should approach a limit of 3.5
It is important that students are encouraged to try and discover and prove any rules.

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It is probably best to explain orally to students how to use the calculator to simulate dice before starting the activity. If the calculator is set to STAT mode, it is possible to store the rolls of the die and use the calculator to find the mean score. It is important to note that the calculator should be set back to floating decimal place mode when calculating the mean score. Alternatively, students could work in pairs using one calculator to simulate the die and another calculator to store the data.

Further Ideas

- Investigate the mean scores obtained by rolling a 1 – 8 die, or a 1 – 12 die, etc.

- Investigate the mean scores obtained by adding the scores on two dice, or adding the scores on three dice. For example, adding two dice may be simulated on the calculator by:

  (1 + RANDOM(5)) + (1 + RANDOM(5)).

Again & Again

High school

Objective

Explore number patterns arising from a variety of situations.
Interpret, generalize and use simple relationships, and generate rules for number sequences.

Explanation of the activity

Express simple functions initially in words and then symbolically.
This activity develops students' understanding of convergent sequences and the concept of a limit.
For more able students, attempting to prove the results will give practice in algebraic manipulation.
This can also be used to introduce an iterative method for solving quadratic equations.

Using the calculator

Calculator functions used: Addition, subtraction, multiplication, division,
last answer memory

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Let's see what happens when we repeat the same equation over and over.

Think of a number.
Divide it by 2 and add 1.

Example:

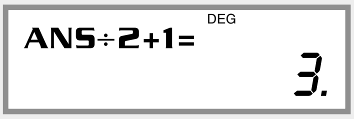
Try the number 6.

6 ÷ 2 + 1 =

6÷2+1= DEG 4.

Divide the answer by 2 and add 1.

÷ 2 + 1 =

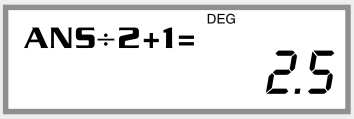


ANS÷2+1= DEG 3.

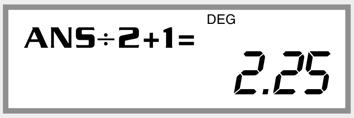
Keep on repeating this.

(÷ 2 + 1) =

=



ANS÷2+1= DEG 2.5



ANS÷2+1= DEG 2.25

⋮

What eventually happened?
Did the answer come closer and closer to a certain integer?

Try investigating different starting values and different rules to use for the repeat calculation.

Again & Again

High school

• • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • •

This activity could be introduced orally. Ask students to think of a number and allow them to generate the sequence using this number as the first term.
Students can investigate different starting values, discovering that all sequences will converge to 2.
More able students could be asked to prove this, i.e.:

$x \div 2 + 1 = x$
$x + 2 = 2x$
$2 = 2x - x$
$2 = x$

Students can investigate different rules such as ÷ 3 + 1, or ÷ 2 + 5, generalizing which rules converge to an integer limit.
Generally, the sequence $\div\, a + b$, converges to $ab/(a - 1)$, where $a \neq 1$ and $a \neq 0$.

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It may be useful to show students how to generate sequences on the calculator.

Further Ideas

- Students use their calculators to generate different convergent sequences. They then have to try and identify the rules that each other used.

- Find rules to define sequences that converge to a given limit. For example, find different sequences that converge to 3.

Fibonacci

High school

Objective

Appreciate the use of letters to represent variables.
Explore number patterns arising from a variety of situations.
Interpret, generalize and use simple relationships, and generate rules for number sequences.
Express simple functions symbolically.

Explanation of the activity

Use the calculator to generate the Fibonacci sequence.

While working on this activity, students will be developing skills of generalization and refining their methods of expressing mathematical rules.

Using the calculator

Calculator functions used: Addition, subtraction, multiplication, division,
last answer memory, Multi-line Playback

The Italian mathematician Fibonacci discovered this sequence:
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

To generate a similar sequence, begin with two numbers and add them to get the next. Continue adding the last two numbers to get the next term.

Let's try making a regular series similar to the Fibonacci series above. First, select two suitable integers.

Press the following buttons and then start operation.

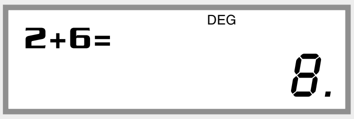
[ON/C] [2ndF] [MODE] [0]

Example:

Let's try making our own similar sequence,
starting with 2 and 6.

[2, 6, ?…]
Adding the two starting numbers yields the third value in the series.

2 [+] 6 [=]



2+6=
DEG
8.

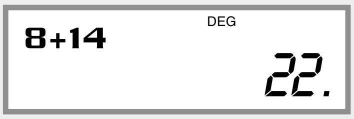
[2, 6, 8, ?…]
The next value is the sum of the two terms preceding it. Thus…

6 [+] 8 [=]

6+8
DEG
14.

[2, 6, 8, 14, ?…]
Similarly, find subsequent terms in the series by adding the preceding term and the one before it.

8 [+] 14 [=]



8+14
DEG
22.

Fibonacci

High school

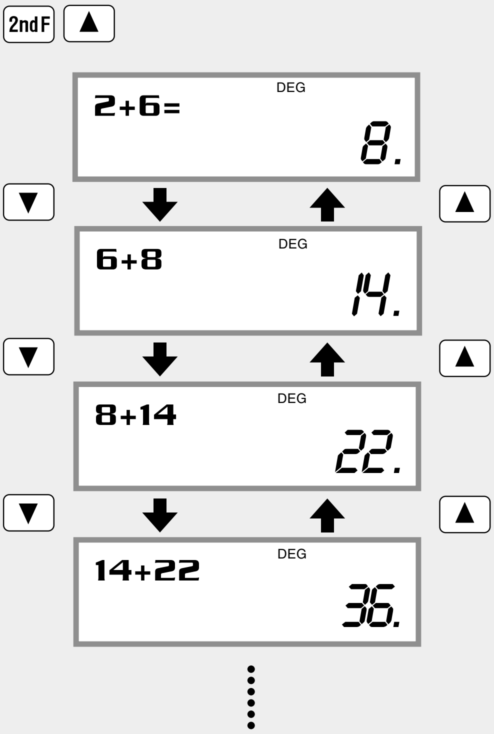
[2, 6, 8, 14, 22, ?…]
Find additional members of the series by repeating the above operation.

[2, 6, 8, 14, 22, 36, 58…]
This yields the series:

Recall the first formula

Try generating similar series using a variety of other numbers.

For each of your sequences, once you have generated a large number of terms, divide the last term by the preceding term and note the answer.



Using the activity in the classroom

Introduce the sequence to the students and give them the opportunity to try to generate it on their calculators. After some discussion the students can be shown how to use the temporary memories to generate the sequence. One method of introducing this is to stick two envelopes onto the board, one labeled 'X' and the other labeled 'Y', in which can be placed pieces of paper marked with the appropriate numbers.
The ratio of successive terms of any such sequence converges to the 'golden ratio', $(1 + \sqrt{5}) \div 2 \approx 1.61803399$.

Points for students to discuss

Explain orally to students how to generate the Fibonacci sequence on the calculator.

Further Ideas

Ask students for two starting values and then generate the first ten terms of the sequence. Predict the sum of these of ten numbers, and then ask the students to confirm your answer. (The 'trick' is that the sum of these numbers will always be the sixth number multiplied by 11.)

Factorizing Quadratics

High school

Objective

Evaluate formulas and expressions.
Manipulate algebraic expressions.

Explanation of the activity

Confirm the equation $x^2 + 5x + 6 = (x + 2)(x + 3)$ is always true by plugging in values for x.

While working on this activity, students will be learning about the factorization of quadratic expressions. Some students may also study the graphs of factorized quadratic functions.

Using the calculator

Calculator functions used: Addition, subtraction, multiplication, division, squaring, memory calculation, Multi-line Playback

Press the following buttons and then start operation.
(Variable x printed in blue under one of keys)

[ON/C] [2ndF] [MODE] [0]

Example:

Confirm the equation $x^2 + 5x + 6 = (x + 2)(x + 3)$ is always true.

(1) Store any number, x, into your calculator.
(2) Use your calculator to evaluate $x^2 + 5x + 6$
(3) Now use your calculator to evaluate $(x + 2)(x + 3)$

Store 14.

14 [STO] [x]

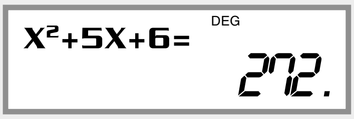
14⇒X

DEG

14.

Calculate $x^2 + 5x + 6$.

[2ndF] [ALPHA x] [x^2] [+] 5 [2ndF] [ALPHA x] [+] 6 [=]

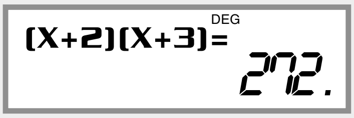


X²+5X+6=

DEG

272.

Calculate $(x + 2)(x + 3)$.



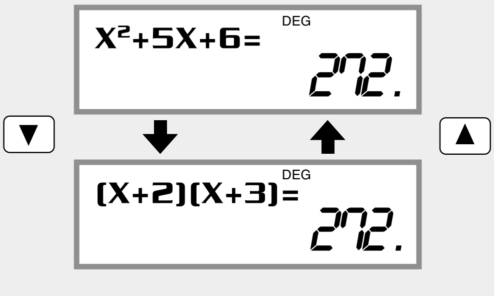
(X+2)(X+3)=

DEG

272.

[(] [2ndF] [ALPHA x] [+] 2 [)] [(] [2ndF] [ALPHA x] [+] 3 [)] [=]

Check with Multi-Line Playback



X²+5X+6=

DEG

272.

(X+2)(X+3)=

DEG

272.

Repeat for other values of x until you are sure the equation always holds true.

Factorizing Quadratics

High school

Use the calculator in the same way to find similar equations for the expressions below.

$x^2 + 7x + 10$
$x^2 + 7x + 12$
$x^2 + 6x + 8$
$x^2 + 10x + 16$
$x^2 + x - 6$
$x^2 + 5x + 14$

• • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • •

It is assumed that students will have done some work on quadratic functions prior to this activity. For example, students may have been attempting to solve a range of quadratic equations given in a variety of forms. Some of these equations could possibly be solved via inverse relationships, (e.g. $x^2 - 4 = 15$), while some may be already given in factorized form and so could be solved directly. Others may have to be solved by trial and improvement. This could lead to students concluding that factorizing is a useful technique in solving quadratic equations.

The activity could then be introduced as an investigation into factorizing quadratic expressions. The teacher could do an example on the board, and then other expressions could be given to the students to factorize, using trial-and-improvement methods on their calculator.

It is important that students are encouraged to reflect on the factorized expressions and look for strategies to help them factorize quadratics. These strategies should be collated together and discussed.

• • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • •

It may be useful to go over with some students how to enter and evaluate algebraic expressions on the calculator.

Further Ideas

The idea can be adapted to focus on the equivalence of other algebraic expressions, including factorizing linear expressions such as $4x + 6$.

Triples

High school

Objective

Understand and use Pythagoras' theorem.
Use coordinate systems to specify location.

Explanation of the activity

Working on this activity will reinforce students' understanding of Pythagoras' theorem, and develop their knowledge of Pythagorean triples. It will also develop their appreciation of different ways of specifying position, including rectangular Cartesian coordinates and polar coordinates.

Using the calculator

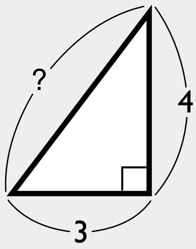
Calculator functions used: Coordinate conversion

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

DRG (Repeat until the DEG symbol appears)

<Example using the EL-531RH>
Try to find the length of the hypotenuse (the side opposite the right angle) of a right triangle whose other two sides are 3 cm and 4 cm long.



Input the lengths of the two shorter sides.

3 2ndF , 4

Find the length of the hypotenuse.

2ndF →rθ

We find that the length of the hypotenuse is 5 cm.

Extra:
Try to find the angle of the corner between the hypotenuse and the base.

2ndF ←,→

We find that this angle is 53.13010235°.

Let's find the hypotenuses of right triangles having various bases and heights.

Triples

High school

• • • • • • • • • • • • Using the activity in the classroom • • • • • • • • • • • •

This activity assumes that students have done some previous work on Pythagoras' theorem. After a short introduction, the students can be set the task of trying to find right-angled triangles where the sides are all integers. The students can be shown how to use the rectangular-polar coordinate conversion function of the calculator to find the hypotenuse given the two shorter sides. It is important that it is explained to students that the calculator is converting a pair of rectangular Cartesian coordinates to polar coordinates, which comprise a distance and an angle.
Students should be encouraged to identify the different 'families' of right-angles triangles, (e.g. 3, 4, 5 and 6, 8, 10 etc.,) and concepts of similarity should be discussed. It is important that students' generalizations are collated and discussed.

• • • • • • • • • • • • • • • • Points for students to discuss • • • • • • • • • • • • • • •

To generate a Pythagorean triple, square any odd number and split the answer into two consecutive integers. The original number and the two consecutive integers will form a Pythagorean triple. For example: $5^2 = 25 = 12 + 13$. Alternatively, square any even number, divide it by two, and then split it into two 'close' integers. The original number and the two 'close' integers will form a Pythagorean triple. For example: $6^2 = 36; 36 \div 2 = 18 = 8 + 10$.

Further Ideas

- When are the areas and perimeters of right-angled triangles numerically equal? Investigate.

- A consideration of the angles obtained by converting rectangular Cartesian coordinates to polar coordinates could form the basis of an introduction to trigonometry.

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